

# Phononic crystal with low filling fraction and absolute acoustic band gap in the audible frequency range: A theoretical and experimental study

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The propagation of acoustic waves in a two-dimensional composite medium constituted of a square array of parallel copper cylinders in air is investigated both theoretically and experimentally. The band structure is calculated with the plane wave expansion (PWE) method by imposing the condition of elastic rigidity to the solid inclusions. The PWE results are then compared to the transmission coefficients computed with the finite difference time domain (FDTD) method for finite thickness composite samples. In the low frequency regime, the band structure calculations agree with the FDTD results indicating that the assumption of infinitely rigid inclusion retains the validity of the PWE results to this frequency domain. These calculations predict that this composite material possesses a large absolute forbidden band in the domain of the audible frequencies. The FDTD spectra reveal also that hollow and filled cylinders produce very similar sound transmission suggesting the possibility of realizing light, effective sonic insulators. Experimental measurements show that the transmission through an array of hollow Cu cylinders drops to noise level throughout frequency interval in good agreement with the calculated forbidden band.

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## I. INTRODUCTION

Elastic analogs of photonic band gap materials [1] have received renewed attention recently. In spite of this analogy, the so-called elastic band gap (EBG) materials need further developments in light of their potential applications in a wide range of technologies. EBG materials, also named phononic crystals, are inhomogeneous elastic media composed of one- [2,3], two- [4,5], or three- [6,7] dimensional periodic arrays of inclusions embedded in a matrix. These composite media typically exhibit stop bands in their transmission spectra where the propagation of sound and vibrations is strictly forbidden. Several classes of EBG materials differing by the physical nature of the inclusions and the matrix have been studied. Among them one finds solid/solid, fluid/fluid, and mixed solid/fluid composite systems. In two-dimensional solid/solid EBG materials composed of periodic arrays of cylindrical inclusions, under the assumption of wave propagation in the plane perpendicular to the cylinders, the vibrational modes decouple in the mixed-polarization modes with the elastic displacement  $\vec{u}$  perpendicular to the cylinders and in the purely transverse modes with  $\vec{u}$  parallel to the inclusions. In contrast, only longitudinal modes are allowed in fluid/fluid composites [8]. The opening of wide acoustic band gaps requires (i) a large contrast in physical properties such as density and speeds of sound, between the inclusions

and the matrix and (ii) a sufficient filling factor of inclusions [8]. In mixed solid/fluid media, the first condition is often satisfied, particularly in the case of solid/gas combinations. The mixed systems present complex vibrational modes ranging from longitudinal modes in the fluid to mixed-polarization modes and transverse vibrations in the solid. In mixed composites, the fluid can be either a condensed liquid (water [9], Hg [10]) or a gas (air [11–17]). The frequency domain where the band gap occurs, scales as the ratio of an effective sound velocity in the composite material to a measure of the periodicity of the array of inclusions. For solid/air systems, the effective sound velocity is significantly lower than that of solid/solid or fluid/fluid composites allowing for the design of acoustic band gaps in the audible frequency domain without excessively large periods and inclusions sizes. In light of this observation, the mixed solid/air EBG materials show the necessary physical characteristics for use as practical sound insulators.

In this paper, we consider a square array of copper cylinders in an air background. The design of such mixed composites presents several difficulties. Theoretically, traditional approaches such as the plane wave expansion (PWE) method fails to predict accurately the acoustic band structures for such a mixed system. This drawback can be alleviated by imposing the condition of elastic rigidity to the solid inclusions [12,13,17]. Within this condition the solid is effectively treated as a fluid. Surprisingly this assumption works reasonably well but does not account for the chemical nature (Cu, steel, W) of the solid nor the geometrical differences such as filled or hollow inclusions. Here we compare the approximate PWE band structure with the transmission coefficients

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where  $\rho(X, Y)$  is the mass density,  $\vec{u}$  and  $\vec{\sigma}$  are the displacement field and the stress tensor. The components of the stress tensor are calculated from the elastic displacement using isotropic Hooke's laws with position dependent elastic coefficients  $C_{11}(X, Y)$  and  $C_{44}(X, Y)$ . The latter elastic constant is zero for a fluid. To calculate the transmission coefficient of a finite size EBG composite, we construct a sample in three parts along the  $Y$  direction, a central region containing the finite phononic crystal sandwiched between two homogeneous regions. A traveling wave packet is launched in the first homogeneous part and it propagates in the direction of increasing  $Y$  across the whole sample. Periodic boundary conditions are applied in the  $X$  direction perpendicular to the direction of propagation. Absorbing Mur's boundary conditions [26] are imposed at the free ends of the homogeneous regions along the  $Y$  direction. The incoming signal is a sinusoidal wave of pulsation  $\omega_0$  weighed by a Gaussian profile and propagates along the  $Y$  direction. In Fourier space this signal varies smoothly and weakly in the interval  $(0, \omega_0)$ . The input signal amplitude does not depend on  $X$ . Space and time are discretized with fine enough intervals to achieve convergence of the finite difference time domain algorithm. Further details concerning the numerical integration of the equation of motion can be found in Ref. [24]. A transmitted signal in the form of the component of the displacement is recorded at the end of the second homogeneous region and integrated along the  $X$  direction. The Fourier transform of that signal normalized to the Fourier transform of a signal propagating through homogeneous material of the same nature as the matrix yields a transmission coefficient.

### III. RESULTS

#### A. Band structure

Figure 2 presents the band structure calculated with the PWE method for a phononic crystal composed of a 2D periodic square array of Cu cylinders of radius  $R = 14$  mm in air. The lattice parameter is  $a = 30$  mm. Calculations were performed considering filled cylinders made of Cu assumed as an infinitely rigid solid. The choice of 1089  $\vec{G}$  vectors of the reciprocal lattice for the computation ensures convergence of the eigenvalues over the range of frequencies studied, i.e., 0–45 kHz. Figure 2 shows unambiguously the existence of absolute stop bands, i.e., band gaps independent of the direction of propagation. The largest observed absolute band gap appears between the first and the second band and extends from 4.2 kHz to 8.4 kHz, which lies in the audible range of frequencies. When considering waves propagating in the direction  $\Gamma X$  of the irreducible 2D Brillouin zone, the lower bound of the local gap occurs at  $\approx 2.8$  kHz. Other local gaps appear at higher frequencies in the directions of propagation  $\Gamma X$  and  $\Gamma M$ . One also notes in both directions of propagation, the existence of relatively flat bands in the band structure. These flat bands are usually associated with the existence of localized states in the composite material [5].

#### B. Computed transmission coefficients

In Figs. 3(a) and 3(b), the computed FDTD transmission coefficients through the 2D square array of filled Cu cylin-

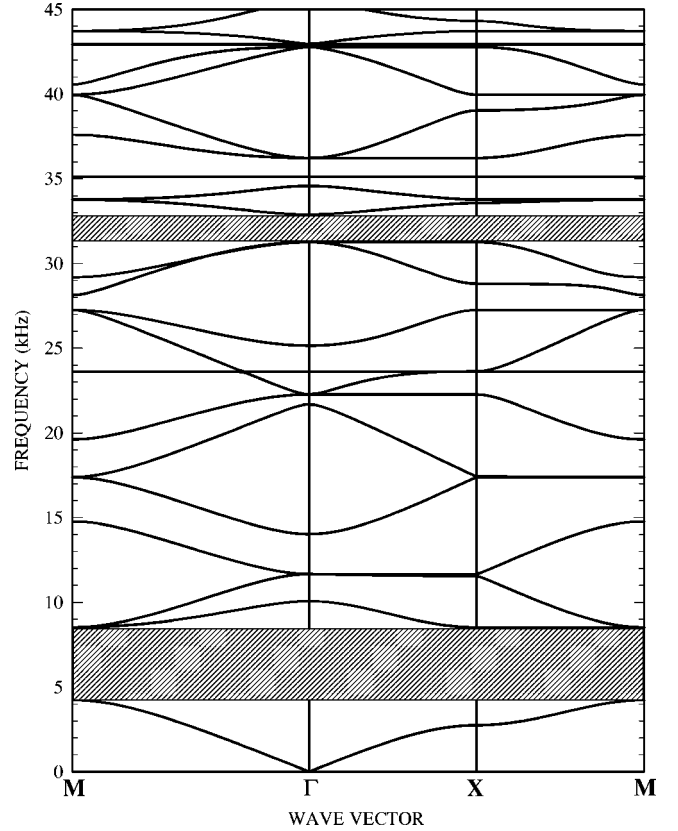


FIG. 2. PWE results for the band structure of the longitudinal modes of vibration in the periodic square array of Cu filled cylinders in air. The radius of the cylinders is  $R = 14$  mm and the lattice parameter is  $a = 30$  mm. The points  $\Gamma$ ,  $X$ , and  $M$  are defined in the inset of Fig. 1. The density,  $\rho$ , and the longitudinal,  $C_l$ , and transverse,  $C_t$ , speeds of sound in air and Cu, are  $\rho^{Air} = 1.3 \text{ kg m}^{-3}$ ,  $C_l^{Air} = 340 \text{ m s}^{-1}$ , and  $\rho^{Cu} = 8950 \text{ kg m}^{-3}$ ,  $C_l^{Cu} = 4330 \text{ m s}^{-1}$ ,  $C_t^{Cu} = 2900 \text{ m s}^{-1}$ . Absolute band gaps are represented as hatched areas.

ders in air along the two principal directions of propagation are presented. These transmission spectra were obtained numerically by solving the equations of motion over  $2^{22}$  time integration steps with each time step lasting 4 ns. The FDTD samples contain six cylindrical inclusions along the  $Y$  direction of propagation. The space is discretized in both  $X$  and  $Y$  directions with a mesh interval of  $10^{-4}$  m. The location and the width of the first absolute band gap in both directions of propagation compare very well with those observed in the band structure of Fig. 2. At higher frequencies, the locations of the local gaps in the  $\Gamma X$  direction overlap in the FDTD spectrum and in the PWE band structure. Moreover one notes that the flat bands observed in the dispersion curves do not contribute significantly to the transmission. Along the  $\Gamma M$  direction, the FDTD transmission spectrum and the PWE band structure lead to rather different results. For instance, a local gap occurs in Fig. 3(b) between 12 and 14 kHz while longitudinal vibrational modes exist in Fig. 2 in this range of frequency. It appears that some of the vibrational modes observed in the PWE dispersion curves do not contribute to the transmission as displayed in the FDTD transmission spectrum. An analysis of the eigenvectors asso-

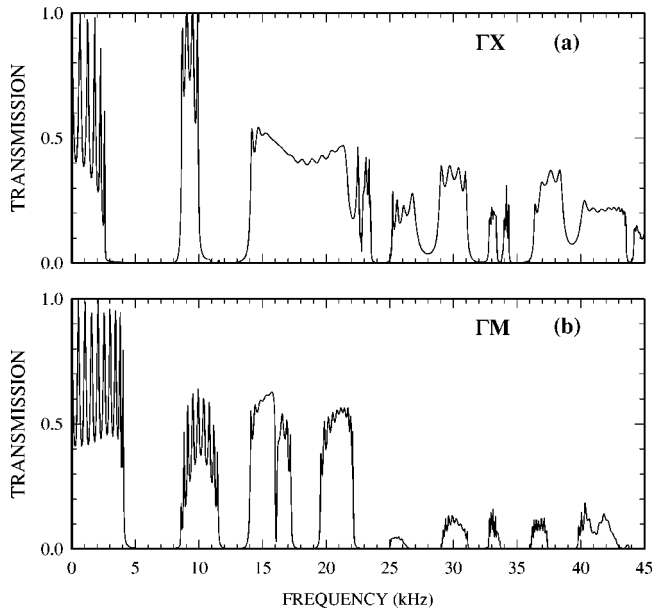


FIG. 3. Transmission coefficient through the square array of filled Cu cylinders in air, computed with the FDTD method along the directions of propagation (a)  $\Gamma X$  and (b)  $\Gamma M$ .

ciated with the different vibrational modes would be helpful for an understanding of these differences between PWE and FDTD results. In both directions of propagation, one also observes a decrease in the amplitude of the transmitted FDTD computed signal on increasing frequencies.

In contrast to the PWE method, the 2D FDTD scheme allows one to distinguish between hollow inclusions and filled cylinders. We have then computed the FDTD transmission coefficients along the  $\Gamma X$  and  $\Gamma M$  directions of propagation through a square array of Cu tubes of inner radius  $R_i=13$  mm and of thickness  $\delta=1$  mm. The lattice parameter is the same as used previously and the FDTD computations were done under the same numerical conditions as those of Figs. 3(a) and 3(b). In particular, with a mesh interval of  $10^{-4}$  m the thickness of the tubes corresponds to ten spatial discretization points. Figures 4(a) and 4(b) present the variation of the computed transmission coefficient as a function of frequency in the directions  $\Gamma X$  and  $\Gamma M$ , respectively, in the range of frequency 0–45 kHz. Except for very slight differences these spectra are surprisingly similar to those obtained with filled cylinders of the same outer radius, i.e.,  $R=14$  mm [see Figs. 3(a) and 3(b)]. This shows that in this range of frequencies where the thickness of the tubes is very much lower than the wavelength of sound and for constituent materials with extremely different physical characteristics such as Cu and air, the thickness of the inclusions does not affect, the transmission of acoustic waves through 2D EBG materials. This theoretical observation agrees with previous experimental results on the transmission of acoustic waves of audible frequencies through square and triangular arrays of hollow and filled stainless steel cylinders in air [16]. However this conclusion is quite dependent upon the choice of the materials constituting the phononic crystal. Therefore, we have considered the case of two-dimensional phononic crystals made of materials whose physical characteristics exhibit

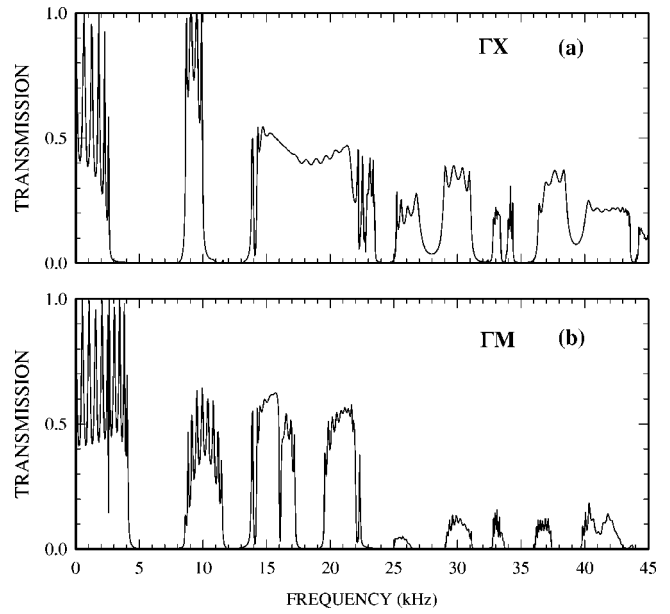


FIG. 4. Same as Fig. 3 but for the square array of Cu tubes of inner radius  $R_i=13$  mm and thickness  $\delta=1$  mm. Air occupies the interior as well as the exterior of the hollow cylinder.

a lower contrast. More specifically, a lower contrast can be obtained by replacing air by water in the two-dimensional phononic band gap material previously studied. Figure 5 shows the FDTD transmission spectra along the  $\Gamma X$  direction of propagation for a square array of filled Cu cylinders immersed in water [see Fig. 5(a)] or hollow Cu inclusions sur-

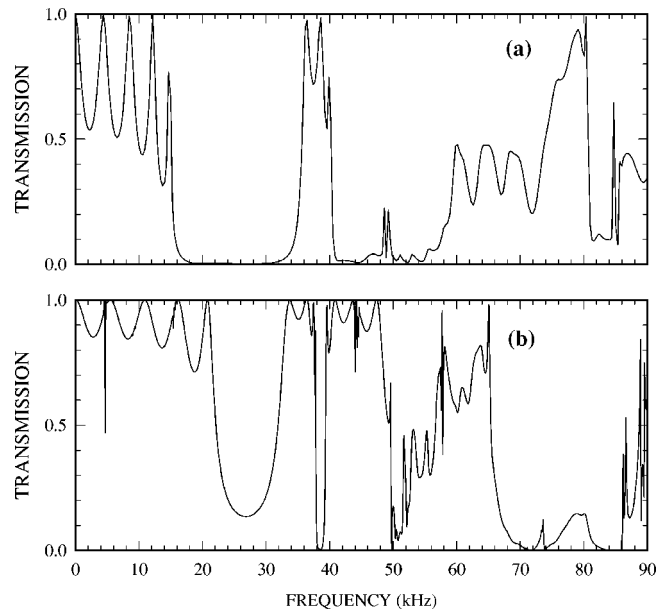


FIG. 5. Transmission coefficient through a square array ( $a=30$  mm) of Cu cylinders in water, computed with the FDTD method along the  $\Gamma X$  direction of propagation for (a) filled cylinders ( $R=14$  mm) and (b) hollow tubes ( $R_i=13$  mm and  $\delta=1$  mm). The density and the longitudinal speed of sound in water are  $\rho=1000$  kg m $^{-3}$  and  $C_L=1490$  m s $^{-1}$ . Water occupies the interior as well as the exterior of the hollow cylinder.



rounded and filled with water [see Fig. 5(b)]. The geometrical parameters were the same as used previously i.e., lattice parameter  $a = 30$  mm, radius  $R = 14$  mm for the filled inclusions, and the inner radius  $R_i = 13$  mm, and the thickness  $\delta = 1$  mm for the tubes. Both spectra have been computed with the same numerical conditions and especially with five cylinders along the  $Y$  direction. These spectra exhibit significant differences in the frequency range of 0–90 kHz. On one hand, Fig. 5(a) shows a very large gap of width 18 kHz centered on 25 kHz while in Fig. 5(b) the transmission just depresses around this frequency. The width of this dip is also smaller than that of the gap observed in Fig. 5(a). On the other hand, at higher frequencies, the transmission spectra are completely different. For example, a gap occurs around 45 kHz in Fig. 5(a) while the transmission for hollow inclusions is maximal in this range of frequency. Another noticeable difference between the two spectra lies in the existence of a zero of transmission at a frequency of 38 kHz in Fig. 5(b). The midfrequency of this small gap depends on the thickness of the hollow cylinders. Indeed a more detailed study shows that the zero transmission frequency may be shifted by changing the thickness of the inclusion. Our FDTD calculations demonstrate clearly that in the peculiar case of Cu/water composite material, the transmission coefficient of acoustic waves is very sensitive to the thickness of the hollow metallic inclusion.

### C. Experimental results

In order to test the theoretical predictions, we have manufactured a phononic crystal composed of a  $10 \times 10$  square array of hollow Cu cylinders. The physical characteristics of the composite material were those considered in the preceding sections, i.e., an inner radius of the tubes  $R_i = 13$  mm, a thickness of the hollow inclusions  $\delta = 1$  mm, and a period of the square lattice  $a = 30$  mm. With this geometry, the filling factor of metallic inclusions is 0.094. It is worth noting that a similar structure built out of filled Cu cylinders would possess a filling factor of 0.684. The tubes of length 450 mm are embedded at one end into a thick steel plate with the other end remaining free. A speaker connected to a low frequencies generator and a microphone are employed to produce an incoming signal and record the transmitted one. The transmitted signal is detected by a tracking generator coupled to a spectrum analyzer. The speaker and the microphone are located 40 mm away from the sample faces. Two measurements are conducted with and without the sample. The difference between the Fourier transforms of both temporal signals is calculated to subtract any background effect. Transmission was measured for acoustic waves in the audible frequency range, perpendicular to the vertical faces of the sample, i.e., along the  $\Gamma X$  direction of propagation. The measured acoustic transmission coefficient of Fig. 6 clearly shows one forbidden band between 4 and 8.8 kHz. The width of this forbidden band is slightly lower than that obtained theoretically from the PWE method and the 2D FDTD scheme [see Figs. 2, 3(a) and 4(a)]. On the other hand, the lower and upper edges of the experimental gap appear at frequencies slightly higher than the predicted ones. This dis-

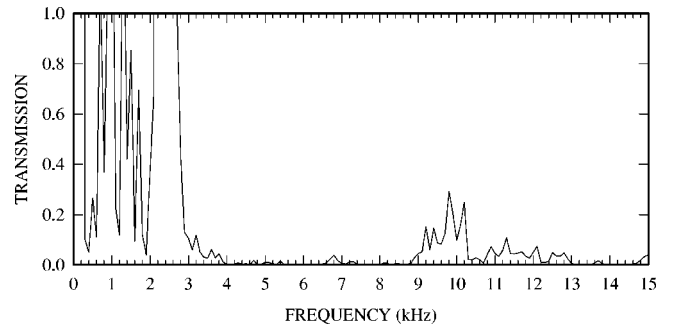


FIG. 6. Transmission coefficient measured perpendicular to the vertical faces of the sample made of  $10 \times 10$  Cu tubes ( $R_i = 13$  mm and  $\delta = 1$  mm) arranged periodically on a square lattice ( $a = 30$  mm).

crepancy between measurements and theoretical predictions may be attributed to the divergence of the emitted acoustic signal, i.e., the fact that the input experimental signal is not a plane wave but is composed of a set of wave vectors inside a cone around the incident direction. In other words, experimentally, the transmission through the sample can occur in a cone around the  $\Gamma X$  direction. At frequencies higher than 8.8 kHz, the transmission is maximal for 9.8 kHz with an amplitude very much lower than that at very low frequencies, i.e., in the range 0–3 kHz. The transmission is then strongly attenuated and it becomes difficult to define precisely the edges of regions with noise level transmission. But these experiments performed with a very usual setup validate fairly well the theoretical predictions concerning the existence of a forbidden band at audible frequencies.

### IV. CONCLUSION

We have investigated theoretically and experimentally the propagation of acoustic waves in a 2D elastic band gap material constituted of a square array of parallel, circular, Cu cylinders in air. The experiments and the theoretical calculations prove the existence of a forbidden band for frequencies in the audible regime. From a theoretical point of view, the comparison between our PWE and FDTD results have shown that the assumption of infinitely rigid solid made for the computation of the band structure is realistic at low frequencies, i.e., for frequencies lower than 10 kHz. At higher frequencies the two theoretical methods give rather different results especially in the  $\Gamma M$  direction of propagation of the irreducible square Brillouin zone. On the other hand, the FDTD method enabled us to differentiate between filled inclusions and hollow tubes. Our FDTD calculations demonstrate undoubtedly that for frequencies in the range 0–45 kHz, filled and hollow metallic inclusions placed in air, lead to very similar transmission coefficients in agreement with other experimental results. In contrary, the transmission coefficient strongly depends on the thickness of the hollow inclusion when air has been replaced by water in the two-dimensional structure. From a practical point of view, the Cu/air composite material, which can be very easily manufactured, is a good candidate for an effective light, sonic

insulator. It should also be possible to shift the forbidden band to much lower audible frequencies by changing the geometry of the array of inclusions.

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